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Abstract

A preeminent argument against efficiency wage theory as an explanation of involuntary unemployment is that a Pareto superior incentive scheme exists which can handle the worker shirking problem: workers could post performance bonds which they forfeit in the event they are caught shirking. The employer moral hazard problem that is created by the possibility of bond expropriation is believed to be solvable by employer reputation. We show that overcoming the employer moral hazard problem by reputation requires extra-normal profits for the firms which in turn imply inefficiencies in the product market rather than in the labor market. Moreover, if firms can enter the product and labor markets freely, an equilibrium with extra-normal profits cannot exist, unless firms have differentiated technical efficiency characteristics. In the latter case, in the process of signalling their types and, thus, establishing their reputations, firms have to follow certain employment paths and to incur costs which, in effect, work as entry barriers and permit market equilibrium to exist. However, if the size of the firm is not observable to workers, again no bonding equilibrium can exist, but an equilibrium with efficiency wages is possible.

I. Introduction

Efficiency wage theory has recently become a popular means of explaining equilibrium unemployment. In a very attractive version of this theory, it is posited that employment relationships suffer from agency problems on the part of workers. The problem of providing appropriate incentives for workers coupled with the inability to perfectly monitor worker effort by firms leads to employment contracts where workers are paid in excess of their reservation wage, but fired if caught shirking. In equilibrium, such contracts necessitate job rationing, giving rise to involuntary unemployment [Shapiro and Stiglitz, 1984; Bulow and Summers, 1986; Yellen, 1984; Katz, 1986].

In spite of its attractiveness both for its relative simplicity and its predictive power, efficiency wage theory has been severely attacked on the grounds that there is a Pareto superior incentive scheme to handle the worker shirking problem: workers could post performance bonds which they forfeit in the event they are caught shirking [Akerlof and Yellen, 1986]. Moreover, an equilibrium with performance bonds is compatible with labor market clearing. Thus, one might quite reasonably ask whether the involuntary unemployment associated with efficiency wage theory is merely an artifact of incorrect modeling. [See the exchange between Carmichael (1985) and Shapiro and Stiglitz (1985).]

Advocates of the efficiency wage view have advanced several reasons why the posting of performance bonds by workers may not be a feasible solution to the agency problem.[1] Among these explanations are liquidity constraints faced by workers, worker reluctance to post performance bonds when there is a risk of being mislabeled as a shirker, and employer moral hazard associated with expropriation of performance bonds, perhaps through the

deliberate mislabeling of workers as shirkers. The preeminent explanation among these appears to be liquidity constraints,[2] though it is somewhat disturbing to have a theory of labor market inefficiencies rely entirely on such a deus ex machina. Similarly, noise in the monitoring of worker effort may contribute to the lack of reliance on performance bonds, but it does not seem to supply an explanation of sufficient weight to rule out performance bonds entirely.

Employer moral hazard problems associated with the expropriation of performance bonds would appear to offer a much more promising explanation for the absence of such bonds in actuality, though it is not our primary aim to provide such an explanation.[3] Instead, our goal is to first explicate the role of firm reputations in sustaining an equilibrium with performance bonds and, second, to address the conventional wisdom that when such an equilibrium exists, it is indeed first best, since there will not be any involuntary unemployment. Our view is that when workers are not liquidity constrained, the use of performance bonds may imply labor market clearing, but this does not in itself imply first best outcomes. When firm moral hazard problems need to be overcome in order for workers to be willing to post bonds, it should not be surprising that the resulting equilibrium is not first best. However, one may have to consider phenomena other than involuntary unemployment to observe the departure from the first best solution.

The source of inefficiencies in our equilibrium is that firms require some incentive to maintain their reputations. Firms need to earn sufficient rents from their reputations that they view expropriation of performance bonds, which causes a loss of reputation, as unattractive. In our model, such rents appear in the form of extra normal future profits.

Our view of firm reputation in the labor market is similar in spirit to some of the recent works concerning firm reputation and the provision of quality in the product market [Klein and Leffler (1981), Shapiro (1982 and 1983), and Allen (1984)]. In particular, our analysis follows Allen's fairly closely. When firm sales are observable by buyers, Allen finds that equilibrium, should it exist, may entail rationing on the part of sellers. That is, price may be in excess of marginal cost. The reason for this rationing is that, were a firm to increase its sales, the firm's customers would no longer find it credible that the firm is providing a high quality product. In this event, these customers would shop elsewhere. In our model, the analogous result is that employers will be rationed in hiring workers, in order to convince the workforce they do employ that the performance bonds will not be expropriated. Thus, in our equilibrium, the marginal value product of labor will exceed the disutility of effort, as in efficiency wage theory, though the labor market will clear.

An important issue with which the product quality literature has grappled is how to reconcile: (i) the fact that firm reputation must generate sufficient rents to resolve the firm moral hazard problem, with (ii) the notion of competitive equilibrium in the long run. Klein and Leffler's solution is to resort to nonprice competition in the form of firm-specific nonsalvageable assets, such as firm logos, which facilitate consumer service. Sunk capacity investment which raises the marginal product of labor curve may serve the same function in the labor market. Thus, the Klein and Leffler approach when applied to the employer moral hazard problem suggests an alternative distortion with regard to factor mix, i.e., firms maintain excess sunk capacity to sustain their reputations in the labor market. Shapiro's (1983) solution is to assume entrants pay

"reputation building fees" in the form of price discounts to their initial customers. We find Shapiro's story quite compelling. However, his model suffers from the criticism that there is nothing which distinguishes established firms from entrants, other than the time of entry. There is no theoretical reason in Shapiro's model why customers should be reluctant to immediately buy from entrants at the price charged by established firms. Allen's (1984) model is sounder from a game-theoretic perspective, but he is forced to utilize sunk fixed costs to support his equilibrium. In the perfectly contestable case, which we focus on, Allen's equilibrium does not exist.

Our goal is to provide a solid theoretical foundation for Shapiro's intuition and thereby to endogenously determine the equilibrium reputation-building fees. In so doing, we borrow heavily from Milgrom and Roberts (1982) and Kreps and Wilson (1982). They show how Selten's chain store paradox can be resolved by considering a model of incomplete information where firm reputation is to be interpreted as a signal of firm type. This view of firm reputation appears to be quite natural and in sharp contrast to the view of reputation in the product quality literature, where firms are intrinsically identical. The reputation-building fees are determined in a signalling equilibrium as a means by which workers are convinced of the firm's type, which in our model is in regard to a technological parameter. Moreover, we obtain a very sharp characterization of the long run equilibrium under some reasonably mild restrictions on the initial beliefs of workers.

Heretofore, we have restricted attention to the case where workers can observe the firm's employment level. Therefore, workers are in a position to verify whether firms earn sufficient rents to deter them from

expropriation of performance bonds. But when workers do not have sufficient information to make such calculations, they will not be willing to post performance bonds.[4] Consequently, when firm employment levels are not observable by workers, an equilibrium with performance bonds is not viable.[5] Thus, it must be the firms who "post the bonds" in this case. This returns us to efficiency wage equilibrium. However, so long as some of the workers' compensation is in the form of deferred payments, there is an additional wrinkle to consider.[6] The employer moral hazard problem does not disappear, but is now referred to as reneging on deferred obligations rather than expropriation of performance bonds. Firms must still earn sufficient rents from their reputations. Yet, when firm employment is not observable, each firm will operate where the marginal value product of labor equals the marginal cost per worker. This solution entails a firm scale which is too large. In this case there is a second type of inefficiency apart from involuntary unemployment.

The remainder of this paper is organized as follows: In Section II we lay out the basic structure of the model for a firm and its employees in the spirit of the 'shirking' model developed by Shapiro and Stiglitz (1984). We demonstrate the nature of the contractual equilibrium viewing the long term contract as a repeated game, as in Bull (1987). We then show that with one type of firm no product market equilibrium, whether firms utilize efficiency wages or performance bonds, can exist. In Section III, we allow for a continuum of firms with different production efficiencies and consider an entry game where firms choose entry fees and their employment paths to signal their types. The entry game is parameterized by the price in the product market. We demonstrate the existence of a long run equilibrium price such that only the most efficient type firm would choose to enter.

Such a firm is indifferent to entering or not because the equilibrium entry fee is just equal to the discounted present value of the extra normal profits the firm would earn if it entered. Moreover, at the long run equilibrium price, a firm with a reputation, i.e. a firm which has already paid the entry fee, operates where the discounted present value of the future profit stream just equals the value of the performance bonds posted by the firm's workers. In this manner the model with a continuum of firm types resolves the nonexistence problem of product market equilibrium discussed in Section II. In Section IV, we provide a discussion of the model when firm employment is not observable by workers. Section V offers a brief conclusion and possible avenues for future research.

II. A Discrete Time Model of Efficiency Wages and Performance Bonds

Consider an economy with three commodities: a produced good, a nonproduced asset which also serves as a numeraire, and a labor-leisure asset. There are two types of agents, workers and firms. Workers may be either employed or unemployed, but are otherwise identical. Each worker is assumed to be infinitely lived; to possess a one period, additively separable, von Neumann-Morgenstern utility function which is linear in both numeraire and leisure consumption; and to be endowed with one indivisible unit of labor-leisure as well as with sufficient amounts of the numeraire that could be posted as the requisite performance bond, should the worker choose to do so. Let e denote worker effort. Employed workers can either shirk ($e = 0$) or provide effort ($e = \bar{e}$). Unemployed workers necessarily take leisure ($e = 0$), and this conveys the same utility payoff as in case of shirking. We normalize the utility function so that when a worker consumes y units of numeraire and expends e units of effort, the utility payoff is $y - e$.

In this section we restrict attention to the case where all firms are technologically identical. Each firm has a production function, f, whose sole argument is the effective labor input. When the workforce of the firm is L and the proportion of the workforce which shirks is k, the output of the firm is given by $f[(1-k)L]$. The function f is assumed to be differentiable, nondecreasing, and to admit U-shaped average and marginal cost curves (measured in effective labor units). In addition, assume the Inada condition, $\lim_{L\to\infty} f'(L) = 0$, and the absence of any fixed costs, $f(0) = 0$. Firms are assumed to be price takers in the product market. The product price is p.

The centerpiece of this section is the employment contract, to which we now turn. Though a worker will typically be employed with a firm for quite some duration, it will be convenient for expository purposes to view the employment contract as a succession of one-period contracts. Such a one-period contract is a triple, (w,B,L), where, should the contract be acceptable to a sufficient number of workers, w is the wage rate to be paid to each worker, B is the performance bond to be posted by each worker, and L is the volume of employment.[7] We will discuss the acceptability requirement further on in this section. For now, simply assume that this requirement is satisfied.

After a contract has been offered and accepted, each worker receives w and posts B. Each worker then chooses to set $e = \bar{e}$ or $e = 0$, and production takes place. Following the efficiency wage literature, we assume that if a worker has chosen to shirk, then there is a probability, θ, that the worker is detected by the firm. The firm learns about worker shirking at the end of the production process, and then chooses H, the number of workers whose performance bonds are to be expropriated. If $\theta kL \leq H$, then all workers

who were detected shirking lose their performance bonds. The probability that any other worker loses his performance bond is $(H-\theta kL)/(L-\theta kL)$. All workers who do not lose their performance bonds get the bonds refunded at the end of the period. Thus, the contract fulfillment issue is a two-sided moral hazard problem. Following the product quality literature, we assume that $f[(1-k)L]$ and BH are observed and, thus, k and H are inferred by all workers in the labor force.[8] Note that in choosing H, the firm bases this choice on k, the proportion of shirkers in its workforce; but in choosing e, a worker cannot base this choice on H, the number of workers whose performance bonds are expropriated. We assume that the detection of both the worker and the firm moral hazards occur prior to the time when the next period's contract is offered. Thus, future firm contract offers, worker acceptance decisions, worker effort choices, and firm expropriation levels can all be made contingent on the current values of k and H.

Our goal is to construct perfect equilibria of the repeated version of this contract game.[9] In fact, we restrict attention to a subclass of these equilibria characterized by two properties: (i) the equilibrium path is stationary and (ii) the punishments for breach of contract are those which have become popular in the efficiency wage and product quality literatures.[10] Thus, the punishment for a worker who is detected shirking is first, forfeiture of performance bond and second, loss of the right of first refusal to the firm's future contract offers, i.e., the worker is fired. The punishment for a firm which has appropriated more performance bonds than justifiable based on detected shirking, $BH > \theta kBL$, is first that no worker will accept a future contract offer from this firm if the contract requires net bonding of the worker, i.e., $w - B$ less than the worker's reservation wage, and second, that if a contract offer has been

accepted, then each worker under contract will shirk.[11] In other words, if the firm has expropriated any performance bonds, it must go out of business. We will demonstrate that these forms of punishment are indeed credible.

Any such equilibrium takes the following form. Let the contract along the equilibrium path be denoted by (w^e, B^e, L^e). As long as the firm has not cheated in any previous period, the firm offers (w^e, B^e, L^e) in the current period. In this event, all workers who are offered the contract accept it. Then all these workers select $e = \bar{e}$ and the firm sets $H = 0$, i.e., there is neither shirking nor bond expropriation in equilibrium. To put the model more in accord with the efficiency wage literature, in particular with Shapiro and Stiglitz (1984), we assume there is a probability, s, that a worker will be separated from the firm for reasons other than shirking. Those workers who were offered the contract in period t and were not separated from the firm get a new offer of the same equilibrium contract in period t + 1. There are $(1-s)L^e$ such workers. The firm replaces the separated workers by hiring new employees from the pool of unemployed workers. There are sL^e new hires. Each new hire also receives the equilibrium contract. Then the process repeats.

Workers can infer the fraction of the firm's workforce which has shirked from its observed volumes of employment and output. It is assumed that the firm will not be found in violation of its contract as long as the volume of performance bonds appropriated is not greater than θkBL. Thus, it is optimal for the firm to appropriate the performance bond of any worker caught shirking. It is also assumed that the firing of workers caught shirking does not constitute a breach of contract by the firm, i.e., fired workers can be readily replaced from the pool of unemployed as long as the firm has not otherwise committed a contract violation. When the equilibrium

contract wage, net of effort costs, is in excess of the workers' reservation wage, the firing of workers caught shirking constitutes the maximum credible punishment that the firm can impose. It is sensible to assume that this is the punishment which the firm utilizes.

We must show that driving the firm out of business is a credible punishment of an employer who has expropriated any bonds. However, before doing this, we consider the worker decision to shirk or not when an arbitrary contract (w,B,L) has been offered and accepted by L workers. It is assumed that the firm will not expropriate any bonds in the current period, and that in the next period the firm will return to the equilibrium contract. To analyze this decision, consider the returns to worker effort as an asset. The effort choice is made to maximize the value of this asset. Let r denote both the market interest rate and the workers' rate of time preference. Let V_U be the expected present value of the discounted utility stream of a worker who is currently unemployed, and V_E the same for a worker who is currently employed under the equilibrium contract. It is easy to see that $V_E = [(w-\bar{e})(1+r)+sV_U]/(r+s)$. Then, the expected value of not shirking given the current contract, V_N, is given by

$$(1) \qquad V_N = w - \bar{e} + \frac{sV_U + (1-s)[RV_E+(1-R)V_U]}{1 + r},$$

where R is the probability of being retained by the firm if the worker is not separated for reasons other than shirking. $R = \min\{1, L^e/[(1-s)(1-\theta k)L]\}$. That is, if firm size shrinks enough, then it is assumed that workers are retained on a pro rata basis. Similarly, the expected value of shirking given the current contract, V_S, is given by[12]

$$(2) \qquad V_S = w + \frac{\theta[V_U-(1+r)B] + (1-\theta)[sV_U+(1-s)(RV_E+(1-R)V_U)]}{(1+r)}.$$

Workers find it optimal to put forth effort as long as $V_N \geq V_S$. This condition is called the no shirking condition (NSC) and is satisfied as long as

$$(3) \qquad \bar{e} \leq \theta\left[B + \frac{(1-s)R(V_E - V_U)}{1+r}\right].$$

Thus, the expected capital loss from being caught shirking, both in terms of forfeiture of the performance bond and in terms of the loss of the right to be offered the equilibrium contract in the next period, must be as great as the utility gain from taking leisure on-the-job.

We can now show that no operation by the firm is a perfect equilibrium. Suppose that indeed along the equilibrium path the firm always offers a contract which is never accepted by any worker. It follows that at any arbitrary contract (w,B,L) which has been accepted and satisfies $B > 0$, the firm should expropriate all the performance bonds, since in the subsequent period the equilibrium path will be re-established, regardless of how much bond is expropriated. Hence, even if the contract has been accepted, the bond will have no incentive effect on the workers' effort decision, since each worker will expect the firm to expropriate all the bond. Moreover, there is no intrinsic value to being employed by the firm because in all future periods the firm is out of business. Therefore, if the contract has been accepted by workers, then each worker will shirk. It follows that if offered such a contract, a worker will only accept it if $w - B \geq rV_U/(1+r)$, i.e., the payment to the worker net of the bond expropriation by the firm is in excess of the worker's reservation wage. But since any worker who accepts such a contract will shirk, the firm cannot make a profit by offering such a contract and, when $V_U > 0$, the firm makes a

loss. Thus, it is optimal to offer a contract which all workers find unacceptable.

Since the no trade equilibrium described above will be played once the firm has expropriated any performance bonds, it is sufficient to consider only the case of full expropriation in analyzing the firm's moral hazard problem. Let the per period profit along the equilibrium path be denoted by π^e; $\pi^e = pf(L^e) - w^e L^e$. Suppose that the firm has offered an arbitrary contract (w,B,L), and this contract has been accepted by L workers. Moreover, suppose that if the firm does not expropriate the bonds in the current period, then the equilibrium contract will be offered, accepted, and fulfilled in all future periods. The firm will fulfill the current contract if the present discounted value of the future profit stream is at least as large as the value of the bonds which the firm could expropriate. This condition is called the no expropriation condition (NEC) and is satisfied as long as

(4) $\pi^e/r \geq (1-k\theta)LB.$

Any contract (w^e,B^e,L^e) which satisfies the NSC, the NEC, $V_E \geq V_U$, and $\pi^e \geq 0$ can be sustained as a subgame-perfect equilibrium. Such equilibria are sustained by workers punishing firms for offering out of equilibrium contracts as well as for expropriating performance bonds. For a given employment level, there may very well exist a family of equilibria, where members of the family differ in the contract wage and performance bond levels, so that greater performance bonds allow for lower efficiency wages while still satisfying the NSC. Obviously, among these equilibria, workers' preferences are opposed to those of firms. It is not our intention to address the bargaining problem implicit in the question of equilibrium

selection. Instead, we merely borrow the assumption from the efficiency wage literature that a firm can extract all the rents from its workers, reflecting the more basic assumption that labor markets are perfectly competitive and therefore the firm's labor supply is perfectly elastic at the reservation wage. We ask whether there are any rents accruing to workers which are embodied in the contract wage, because equilibrium contracts must reconcile the two incentive problems, even though workers are assumed to have no bargaining power.

We address this question by supposing that the firm is only punished if it has expropriated bond. That is, if the firm offers an out-of-equilibrium contract, then the game proceeds henceforth under the assumption that the firm goes unpunished as long as it does not expropriate the bonds in this out-of-equilibrium contract. Call any such equilibrium a breach as expropriation equilibrium (BEE). The main result of this section is that there is an essentially unique BEE. The BEE is either the no trade equilibrium or an equilibrium with full bonding, i.e., $w^e = \bar{e} + rV_U/(1+r)$ and $B^e \geq \bar{e}/\theta$. In the latter case, the only source of non-uniqueness is that redundant bonding is possible when the NEC is not binding. It is important to note that a contract with efficiency wages cannot be a BEE.

Let (w^e, B^e, L^e) be a BEE contract and suppose the firm has made an arbitrary offer (w,B,L) which has been accepted by L workers. The firm does not expropriate the bonds so long as the NEC is satisfied. Hence, all the L workers who were offered the contract choose $e = \bar{e}$ as long as the NSC is satisfied. When the contract (w,B,L) satisfies both the NSC and the NEC, workers who are offered the contract accept it as long as

$$(5) \qquad w - \bar{e} + \frac{sV_U + (1-s)[RV_E + (1-R)V_U]}{1 + r} \geq V_U.$$

Inequality (5) is termed the contract acceptability condition (CAC). The firm will offer a contract which maximizes its profits in the current period subject to the NSC, NEC, and CAC. It follows that in such a profit maximizing contract, the CAC must be binding. To see this observe that the current wage payment does not enter into either the NSC or the NEC. Hence, if the CAC were not binding, the firm could offer an alternative contract which differed from the profit maximizing contract only in that the alternative one had a lower wage, and hence a higher profit. Obviously, such an alternative contract cannot exist.

Since the CAC is binding and the left hand side of (5) is V_N as defined in (1), it follows that $V_N = V_U$. But in a stationary equilibrium $V_E = V_N$. Hence, the equilibrium contract must be a full bonding one.

The reader may be uncomfortable with this result because it is based on the assumption that the firm cannot precommit to a contract path. Indeed, the objection might be raised that the period in which detection of the moral hazard occurs is not coincident with the period in which a new contract offer from the firm is forthcoming. We concur with this objection, viewing it likely that the latter is substantially longer than the former. But the conclusion that equilibrium contracts require full bonding is robust to letting firms have some degree of precommitment in their future contract offerings, though the argument explaining why this is true is different from the intertemporal wage discrimination argument we gave in the no precommitment case.

A straightforward way to understand this other argument is to grant the firm the ability to precommit to the same contract in all periods and assume these precommitted to contracts will be in force as long as the firm has not cheated. Call such an equilibrium a precommitted breach as expropriation equilibrium (PBEE). Then the PBEE NSC is

$$(6) \qquad w^e + \frac{r+s}{1-s} B^e \geq \bar{e} + \frac{r}{1+r} V_U + \frac{r+s}{1-s} \frac{\bar{e}}{\theta}.$$

The PBEE NEC, assuming $L^e > 0$, is[13]

$$(7) \qquad \frac{pf(L^e)}{L^e} - w^e - rB^e \geq 0.$$

Suppose the firm is offering an equilibrium contract which has some degree of efficiency wage payment in it. It follows that (6) and (7) must both be binding. But along the NSC constraint, $dw^e/dB^e = -(r+s)/(1-s)$. Therefore, holding employment constant, the firm could change its contract offer along the NSC constraint by raising B^e and lowering w^e. Such a move slackens the NEC as longs as $s > 0$, and has no impact when $s = 0$.[14] Thus such a move increases profits, because labor costs have been reduced.

When there is employment in the PBEE equilibrium, the PBEE employment level either solves

$$(8) \qquad \max_{L \geq 0} \; pf(L) - [\bar{e} + \frac{rV_U}{1+r}]L,$$

in which case (7) is not binding, or is implicitly given by[15]

$$(9) \qquad \frac{pf(L^e)}{L^e} = \bar{e} + \frac{rV_U}{1+r} + \frac{r\bar{e}}{\theta}.$$

Let $\bar{L}$ be the input level where maximum average product is obtained. If $pf(\bar{L})/\bar{L} < \bar{e} + rV_U/(1+r) + r\bar{e}/\theta$, then the PBEE is necessarily the no trade equilibrium.

We conclude this section by observing that this view of worker-firm contracting is incompatible with product market equilibrium under free entry. There is no value of p for which there is a PBEE with the firm in operation, yet earning zero profit. We attempt to resolve this dilemma in the next two sections.

III. A Model of Entry with a Continuum of Different Firm Types

In this section, we expand the above basic model by allowing for differentiated firms and by assuming that workers cannot observe the type of a given firm. As a result of this information asymmetry, workers cannot verify directly whether a particular firm's contract satisfies the NEC or not. In the equilibrium which is described below, firms signal their types by choosing both an endogenously determined entry fee, which is an increasing function of initial employment, and by pursuing a particular employment path over time. Both choices can be interpreted as acts of reputation building. Consistent with this interpretation, a firm has a reputation if it has paid the entry fee and employed workers for a sufficient number of periods that its type is uniquely identified.

Let $m \in [\underline{m},1]$ denote the firm type, with $\underline{m} > 0$. m refers to a technological parameter which, for simplicity, we take to be a multiplicative factor, so that mf is the production function of a type m firm.[16] It is assumed that no firm can impose punishment on any other firm. Only workers can punish a firm for expropriation of performance bonds and this is accomplished, as discussed in Section II, by refusing to work for this firm in all periods subsequent to the time of bond expropriation. Thus, the BEE contracts described in the previous section may not be sustainable. A type $\tilde{m}$ firm might find it more profitable to offer the BEE contract of a type m firm for one period, expropriate the performance bonds, and go out of business than to offer the BEE contract for a type $\tilde{m}$ firm. As a result of this problem, the type m firm will attempt to make its contract unattractive for the purpose of bond expropriation by other type firms. It does this by judiciously choosing its entry fee and employment path. In the process of deterring other firms from bond expropriation, the firm signals its type.

Without loss of generality, we restrict attention to full bonding contracts. Any contract which calls for workers to be paid in excess of their reservation wage plus effort costs in some periods could be replaced by a full bonding contract with the same employment levels, where the present discounted value of the wage premia are loaded into the entry fee. This alternate full bonding contract provides the firm with identical profits, and if the original contract deterred other firms from expropriating performance bonds, then the alternate contract would do so as well.

In the game that we consider here, each firm attempts to convince its workers that there is no risk of bond expropriation. The equilibria of the game have the properties that no firm has any incentive to expropriate performance bonds and no firm desires to change its contract offer, given the contract offers of the other firms. In order to center in on these properties, we assume that firm strategies are in regard to contract offers only. The firm decision to expropriate performance bonds or not as well as the worker decisions to accept or reject the contract offer and to shirk or put forth effort are all modeled implicitly through specification of the payoff functions of the game. Denote a strategy for a type m firm by $\sigma(m) = [C(m),\{L_t(m)\}]$, where $C(m)$ is an entry fee, $\{L_t(m)\}$ is a sequence of employment levels, and $L_t(m)$ is the employment level $t - 1$ periods after entry has occurred.[17]

The reader will note that $\sigma(m)$ is an open loop strategy. This observation bears some comment, since it is reasonable to ask whether closed loop strategies would be more appropriate. We think not, on the grounds that the added complication would provide little insight and would take us far afield of our main purpose. The only source of strategic interaction between firms is through the convincing of workers that a particular contract offer does

not involve any expropriation risk. Workers are so convinced when it is not optimal for any type firm to offer the contract with the intent of bond expropriation. If workers are able to accurately assess the equilibrium payoffs for each firm type, they can determine whether a particular contract offer involves expropriation risk. The essential difference in this regard between open loop equilibria and closed loop equilibria is that, with the former the workers' assessments of the equilibrium payoffs for each firm are independent of the actual play of the game, while with the latter the workers' assessments may vary with the history of play.

As already mentioned, our equilibria have the property that there is no expropriation risk involved in accepting equilibrium contracts. It is conceivable, however, that workers will accept a contract with expropriation risk if they are paid a compensating differential to offset the expected capital loss due to bond expropriation. We rule out equilibria with such compensating differentials by restricting workers' prior beliefs over firm types. Each worker's initial beliefs over firm types can be represented by a probability distribution on the interval $[\underline{m},1]$. These distributions may vary across workers, but each distribution is required to be nonatomic and with support equal to the entire interval $[\underline{m},1]$. The consequence of this restriction on worker beliefs is as follows. If there is any perception of expropriation risk, then all workers will in fact expect expropriation with certainty. Then the performance bonds will not deter worker shirking and the requisite compensating differential to make the contract acceptable to workers is too large to be economic from the point of view of the firm. The payoff function defined in the next paragraph is constructed with this restriction in mind.

Let $w_R = \bar{e} + rV_U/(1+r)$, $B_F = \bar{e}/\theta$, and

$$\pi(m,\sigma(m)) = \sum_{t=1}^{\infty} \frac{pmf(L_t(m)) - w_R L_t(m)}{(1+r)^{t-1}} - C(m).$$

$\pi(m,\sigma(m))$ is the present discounted value of profits for a type m firm from pursuing the strategy $\sigma(m)$ if that strategy is acceptable. Let the joint strategy for all firm types be denoted by σ,[18]

$$\sigma = \mathop{X}_{m\varepsilon[\underline{m},1]} \sigma(m).$$

The payoff function, denoted by u, maps firm types and joint strategies into profits. The payoff function is defined by

(8) $\quad u(m,\sigma) = \pi(m,\sigma(m))$ if for all $\tilde{m} \varepsilon [\underline{m},1]$ and for all $t = 1,2,3,...$

$$\sum_{h=1}^{t} \frac{p\tilde{m}f(L_h(m)) - w_R L_h(m)}{(1+r)^{h-1}} + \frac{B_F L_t(m)}{(1+r)^{t-1}} - C(m)$$

$$\leq \max[\pi(\tilde{m},\sigma(\tilde{m})),\pi(\tilde{m},\sigma(m))],$$

$u(m,\sigma) = 0$ otherwise.

The constraints in (8) say that no other type of firm $\tilde{m}$ prefers to select the strategy $\sigma(m)$ for the purpose of bond expropriation over selecting the strategy $\sigma(\tilde{m})$. Note that when $\pi(\tilde{m},\sigma(m)) \geq \pi(\tilde{m},\sigma(\tilde{m}))$, these constraints reduce to

(9) $$B_F L_t(m) \leq \sum_{h=t+1}^{\infty} \left[\frac{p\tilde{m}f(L_h(m)) - w_R L_h(m)}{(1+r)^{h-t}}\right],$$

for all t. This is the NEC for a type $\tilde{m}$ firm playing strategy $\sigma(m)$. Thus, workers do not fear the threat of expropriation by type $\tilde{m}$ firms if such firms would prefer to pursue $\sigma(m)$ in its entirety rather than to pursue $\sigma(m)$ only for a finite number of periods and then expropriate the bonds. Though the equilibria of this game will have the property that $\pi(\tilde{m},\sigma(\tilde{m})) \geq \pi(\tilde{m},\sigma(m))$, there is still a reason for leaving the right hand side of (8) as it is rather than putting $\pi(\tilde{m},\sigma(\tilde{m}))$ there instead. This reason is illustrated by assuming instead that this alteration has been made and that $\pi(1,\sigma(1)) = 0$. Then, since the left hand side of (8) is nondecreasing in $\tilde{m}$, by taking t large enough in (8) it is straightforward to show that satisfaction of (8) requires that $\pi(m,\sigma(m)) \leq 0$ for all $\tilde{m} \in [\underline{m},1]$. Thus, a trivial equilibrium with no firm in operation always exists in this case. Our specification of (8) rules out these trivial equilibria as is shown by Lemma 1 below.

Given the payoff functions, Nash equilibrium is defined in the usual way.[19] In such a Nash equilibrium there is a critical firm type, m_c, such that for all $m > m_c$, type m firms earn strictly positive profits in equilibrium and for all $m < m_c$, type m firms do not operate and hence earn zero profits in equilibrium. We wish to rule out the case where $m_c = \underline{m}$. In order to do so we assume

$$(10) \qquad \frac{p\underline{m}f(\overline{L})}{\overline{L}} - w_R < rB_F, \quad \text{while } \frac{pf(\overline{L})}{\overline{L}} - w_R \geq rB_F.$$

In other words, the least efficient type firm cannot satisfy the NEC while the most efficient type firm can satisfy the NEC if it offers a stationary employment contract at the employment level which maximizes average product, or equivalently, profits per worker employed in the firm. m_c is then determined by

(11) $$\frac{pm_c f(\overline{L})}{\overline{L}} - w_R = rB_F, \quad \text{or} \quad m_c = \frac{rB_F + w_R}{pf(\overline{L})/\overline{L}}.$$

<u>Lemma 1</u>: Consider the strategy $\hat{\sigma} = (C,\{L_t\})$ where $C = (1+r)B_F\overline{L}$ and $L_t = \overline{L}$ for all t. If $m > (rB_F+w_R)\overline{L}/[pf(\overline{L})]$, then

$$B_F L_t = B_F\overline{L} < \frac{pmf(\overline{L}) - w_R\overline{L}}{r} = \sum_{h=t+1}^{\infty} \frac{pmf(L_h) - w_R L_h}{(1+r)^{h-t}},$$

and

$$\left(\frac{1+r}{r}\right)[pmf(\overline{L})-(w_R+rB_F)\overline{L}] = \sum_{t=1}^{\infty} \frac{pmf(L_t) - w_R L_t}{(1+r)^{t-1}} - C > 0.$$

While if $m < (rB_F+w_R)\overline{L}/[pf(\overline{L})]$, then

$$pmf(L_1) + (B_F-w_R)L_1 - C = pmf(\overline{L}) + (B_F-w_R)\overline{L} - (1+r)B_F\overline{L} < 0$$

and

$$\sum_{h=1}^{t} \frac{pmf(L_h) - w_R L_h}{(1+r)^{h-1}} + \frac{B_F L_t}{(1+r)^{t-1}} - C$$

$$= \frac{(1+r)^t-1}{r(1+r)^{t-1}}\,[pmf(\overline{L})-w_R\overline{L}] + \frac{B_F\overline{L}}{(1+r)^{t-1}} - (1+r)B_F\overline{L} < 0$$

for all $t = 2,3,\ldots.$

<u>Proof</u>: Trivial.

Lemma 1 implies that $m_c \leq (rB_F+w_R)\overline{L}/[pf(\overline{L})]$, since for any $m > (rB_F+w_R)\overline{L}/[pf(\overline{L})]$, a type m firm could always choose $\hat{\sigma}$, satisfy the constraints in (8), and make positive profits. On the other hand, $m_c < (rB_F+w_R)\overline{L}/[pf(\overline{L})]$ is not possible since satisfaction of (9) is not

possible for any contract with positive employment when $m < (rB_F + w_R)L/[pf(L)]$. This result is proven in Lemma 3.

We turn now to the consideration of the best response functions for firms of type $m \geq m_c$. From (8) it follows that these best responses depend on other firms' strategies only insofar as these strategies determine firm profits. Let L^* maximize the per-period profit for the most efficient type firm; i.e., $pf'(L^*) = w_R$. Let $z\colon [\underline{m},1] \to [0,(pf(L^*)-w_R L^*)(1+r)/r]$ be such that $z(m) = 0$ for $m \leq m_c$, z is continuous, and z is nondecreasing. (In fact, we can take z to be convex as shown by Lemma 3 below.) z assigns profits to firm types. Note that we require profits assigned to type m firms for $m \in [\underline{m},m_c)$ to be equal to zero. Such firms earn zero profit by not participating. Moreover, the left hand side of (8) is increasing in $\tilde{m}$. Since we require $z(m_c) = 0$, satisfaction of (8) for $\tilde{m} = m_c$ implies satisfaction of (8) for $\tilde{m} = m$ with $m < m_c$.

For $m \in [m_c,1]$, a type m firm solves the following problem:

$$(12) \qquad \underset{\sigma(m)}{\text{maximize}}\ \pi(m,\sigma(m))$$

subject to:

for all $t = 1,2,3,\ldots$ and $\tilde{m} \in [\underline{m},1]$,

$$\sum_{h=1}^{t} \frac{p\tilde{m}f(L_h(m)) - w_R L_h(m)}{(1+r)^{h-1}} + \frac{B_F L_t(m)}{(1+r)^{t-1}} - C(m) \leq \max[z(\tilde{m}),\pi(\tilde{m},\sigma(m))].$$

<u>Lemma 2</u>: There exists a solution to (12) for each $m \in [m_c,1]$.

<u>Proof</u>: The proofs of Lemma 2 and all the remaining propositions in the paper are provided in the Appendix.

Let $\sigma^*(m,z)$ denote a solution to (12) and let $\rho(m,z) = \pi(m,\sigma^*(m,z))$ for $m \in [m_c,1]$.

Lemma 3: $\rho(m,z)$ is increasing and convex in m, and $\rho(m_c,z) = 0$.

We are now ready to prove existence of Nash equilibrium for the entry game. Our emphasis on the properties of the best response payoff function ρ rather than on the best response correspondence itself stems from the fact that the constraint set in (12) may not be convex and hence the optimum correspondence need not be convex valued. We can nevertheless demonstrate the existence of a profit assignment function, z*, such that $\rho(m,z^*) = z^*(m)$ for $m \in [m_c,1]$. Any best response to z* will then yield a Nash equilibrium of the game.

Theorem 1: There exists a profit assignment function z* such that $\rho(m,z^*) = z^*(m)$ for $m \in [m_c,1]$.

Henceforth let σ^* denote a Nash equilibrium. We show that σ^* is characterized by two properties. First, among those types that enter, σ^* separates types. That is, $\sigma^*(m) \neq \sigma^*(\tilde{m})$ for $m, \tilde{m} \geq m_c$ and $m \neq \tilde{m}$. Second, let L*(m) denote the BEE employment level of a type m firm. Then either there exists $T(m,\sigma^*(m))$ such that $L_t^*(m) = L^*(m)$ for $t \geq T(m,\sigma^*(m))$, or $L_t^*(m) < L^*(m)$ for all t and $\lim_{t\to\infty} L_t^*(m) = L^*(m)$. We shall refer to $\sigma^*(m)$ in the former case as a reputation building strategy and in the latter case as an asymptotic reputation building strategy.

Let the strategy constructed from $\sigma(m)$ but stationary from period T onwards be given by $\sigma^T(m) = [C^T(\sigma(m)), \{L_t^T(\sigma(m))\}]$, where $C^T(\sigma(m)) = C(m)$, $L_t^T(\sigma(m)) = L_t(m)$ for $t = 1,\ldots,T-1$, and $L_t^T(\sigma(m)) = L_T(m)$ for $t \geq T$.

<u>Lemma 4</u>: Let z^* be the profit assignment function associated with σ^*. That is, $z^*(m) = \pi(m,\sigma^*(m))$ for $m \in [m_c,1]$ and $z^*(m) = 0$ for $m \in [\underline{m},m_c)$. Then, given z^*, $\sigma^{*T}(m)$ satisfies (12i) and (12ii) for each T and for every $m \in [m_c,1]$.

<u>Lemma 5</u>: $L_t^*(m) \leq L^*(m)$ for each t and for every $m \in [m_c,1]$.

<u>Note</u>: It follows from Lemmas 4 and 5 that either $\sigma^*(m)$ is a reputation building strategy or $L_t^*(m) < L^*(m)$ for each t.
We characterize the Nash equilibria in theorems 2 and 3.

<u>Theorem 2</u>: If $m,\tilde{m} \in [m_c,1]$ and $m \neq \tilde{m}$, then $\sigma^*(m) \neq \sigma^*(\tilde{m})$.
Theorem 2 says that among these types that do enter, the equilibrium separates types.

<u>Theorem 3</u>: Suppose $m_c < m \leq 1$. If $\sigma^*(m)$ is not reputation building, then $\sigma^*(m)$ is asymptotic reputation building.

Having shown existence of Nash equilibrium in Theorem 1 and having characterized such equilibria in Theorems 2 and 3, we feel that it is appropriate to designate the situation where $m_c = 1$ as the long run. In this case, the only type of firm which can successfully enter is the most efficient one, and such a firm is indifferent to enter or not. Note that the strategy $\hat{\sigma}$ is optimal for a type m_c firm so that in the long run entry can be interpreted as requiring the payment of an entry fee equal to $B_F\overline{L}(1+r)/r$ whereupon firm reputation is immediately established. The long run equilibrium price is $p^* = (rB_F+w_R)\overline{L}/f(\overline{L})$, which clearly exceeds the social unit cost of production when there is no agency problem in the labor market, i.e., $w_R\overline{L}/f(\overline{L})$.

Note that so far we have considered V_U--and, therefore, $w_R = \bar{e} + rV_U/(1+r)$--as parameters. It is certainly correct to take V_U fixed in analyzing the equilibrium of an individual firm or of a small sector of an economy. However, in analyzing the equilibrium of the economy as a whole, the expected present value of a currently unemployed worker's utility is endogenous, determined by the job opportunities and wage rates offered in the economy. To see how V_U is set in the long run equilibrium, assume that there are N workers in the economy and that M firms satisfy the aggregate demand for the product at price p*. If total employment, $M\bar{L}$, is less than N, then $M\bar{L}$ - N workers will be voluntarily unemployed and indifferent between taking a job or not. In this case, $V_U = 0$, independently of equilibrium price and employment, and $w_R = \bar{e}$. However, if $M\bar{L} = N$, then output supply is restricted to $(N/\bar{L})f(\bar{L})$, and the product price has to be high enough to equilibrate the supply and demand in the product market. The wage rate may also have to be higher to equate the demand for labor to N. Thus, the wage rate may be higher than the disutility of effort, in which case all workers strictly prefer to be employed than to taking leisure off the job. Since in such an equilibrium everyone is employed, the reservation wage of workers is equal to their market wage rate, $p^*f(\bar{L})/\bar{L} - rB_F$, which exceeds their disutility of effort. Therefore, V_U in this case is equal to $[p^*f(\bar{L})/\bar{L} - rB_F - \bar{e}](1+r)/r$.

Note that when s = 0 and $V_U = 0$, the long run equilibrium price in this bonding equilibrium coincides with the long run equilibrium price obtained in a Shapiro-Stiglitz efficiency wage equilibrium and in both models firms operate at $\bar{L}$. In this case the efficiency wage and bonding equilibria are Pareto equivalent, abstracting from any consideration of income distribution. When s > 0, the long run price in the efficiency wage equilibrium

will be higher than the corresponding price in the bonding equilibrium for two reasons. First, in the presence of turnover, the efficiency wage has to rise to compensate for the shortened expected employment horizon (see note 14). Second, turnover provides the possibility of reemployment for unemployed workers, thus raising V_U in an efficiency wage equilibrium where workers are involuntarily unemployed.

IV. The Model When Employment Is Unobservable

In this section, workers are taken to be unable to observe firm employment. The consequence of this assumption is that performance bonds are ruled out entirely, because if workers were willing to post performance bonds, then a firm could operate for one period at an arbitrarily large scale with the sole intention of expropriating the bonds. Obviously, this cannot occur in equilibrium. As a result, either firms must pay efficiency wages or firms, rather than workers, must "post the bonds" in equilibrium contracts. Efficiency wage contracts represent a perfect equilibrium in this case, because if production with efficiency wages is profitable in one period, it will be profitable in all periods and, therefore, employed workers can rationally expect a future stream of premiums over their reservation wages as long as they are not separated from the firm.

Bond posting by firms differs from efficiency wage contracts in that it involves a form of deferred payments to workers (see note 6).[20] This reintroduces the employer moral hazard problem which needs to be overcome by mechanisms such as the one analyzed in Section II. For simplicity, let us restrict contracts to be of the following form, (w,S), where w is the contract wage and S is a severance payment made only in the event that the worker is separated for reasons other than shirking. The severance payment

is received by the worker in the period subsequent to separation. Then

$$V_E = \frac{(1+r)(w-\bar{e}) + s(S+V_u)}{r+s}, \text{ and the NSC is}$$

(13) $$w - \bar{e} + \frac{s(S+V_U) + (1-s)V_E}{1+r} \geq w + \frac{\theta V_u + (1-\theta)\{s(S+V_U) + (1-s)V_E\}}{1+r}.$$

Substituting in for V_E yields

(14) $$[\frac{r+s}{\theta} + 1-s]\bar{e} + \frac{r(1-s)}{1+r} V_U \leq sS + (1-s)w.$$

Firms will offer contracts where (14) binds. Then substituting s back into V_E results in

(15) $$V_E = w - \bar{e} + \frac{\bar{e}}{\theta} + \frac{V_U}{1+r}$$

Among those contracts where the NSC binds, V_E is increasing in w. Thus, the firm has an incentive to load the bond into the severance payment in order to extract rents from its workers. On the other hand, it is necessary that $S + V_U \leq V_E$, to avoid excessive turnover. This quit moral hazard constraint restricts the firm's ability to extract rents from its workers. In an optimal contract both the NSC and the quit moral hazard constraint bind, yielding optimal contract values

(16) $$w^e = \bar{e} + \frac{r}{1+r} V_U + \frac{r\bar{e}}{\theta}, \text{ and}$$

$$S^e = (1+r)\frac{\bar{e}}{\theta}.$$

Note that it is optimal for the firm to post the full bond, $\bar{e}/\theta$, for the worker. Interest on this bond, $r\bar{e}/\theta$, is paid as part of the contract wage as long as the worker is employed by the firm. The reason for doing this

is that were the interest on the bond not paid to the worker, the worker would be better off quitting and getting hold of the severance payment as soon as possible. Note further that in the equilibrium with severance payments $V_E - V_U = (1+r)\bar{e}/\theta$. Thus, the case of performance bonds posted by firms does not preclude involuntary unemployment though, at fixed V_U, the wedge between V_E and V_U will be less than in the case where efficiency wages are used exclusively. In the latter case, this edge is given by $V_E - V_U = (1+r)\bar{e}/[\theta(1-s)]$. The reason for this difference is as follows. Current wage payments as well as all past wage payments have no incentive effects on the workers' effort decision, and in this sense can be thought of as sunk costs. By deferring some of the workers' compensation, the sunk costs associated with a worker who is separated in period t are less than the analogous sunk costs incurred when efficiency wages are utilized.

A type m firm which plans to honor its contract involving severance payments will choose the employment level to solve

$$\text{(17)} \qquad \underset{L\geq 0}{\text{maximize}}\ pmf(L) - [w^e + sS^e/(1+r)]L.$$

At an interior solution of this maximization problem, the first order necessary condition is

$$\text{(18)} \qquad pmf'(L) = w^e + sS^e/(1+r).$$

A firm could go out of business to renege on its severance payment obligations. In order for contract fulfillment to be optimal, it is necessary that[21]

$$\text{(19)} \qquad pmf(L)/L - [w^e + sS^e/(1+r)] \geq sS^e[r/(1+r)],$$

or

$$(19') \qquad pmf(L)/L \geq w^e + sS^e.$$

This condition is the analogue of the no expropriation condition in the case of observable employment. Note that when firm employment is not observable, aggregate shirking, and therefore the firm's reneging on its severance payment obligations, cannot be inferred from the observed output level as it was assumed possible in Section III. In this case, we assume that a firm's breach of contract becomes known to all workers in the economy by "word-of-mouth."

By subtracting (18) from (19') one obtains

$$(20) \qquad pm[f(L)/L-f'(L)] \geq sS^e[r/(1+r)].$$

In the equilibrium of the entry game, all firms which enter pay the same entry fee, C.[22] This entry fee equals the profits obtained from in and out entry by the least efficient firm which enters. That is,

$$(21) \qquad C = pm_c f(L(m_c)) - w^e L(m_c),$$

where m_c denotes the type of the least efficient firm which enters and $L(m_c)$ solves (17) for $m = m_c$. m_c is implicitly determined where (19') holds as an equality for $m = m_c$ and $L = L(m_c)$.

As in the previous section, we define the long run equilibrium price to be determined where $m_c = 1$. We note that (18) requires $L > \overline{L}$. Thus, in this equilibrium there are two distinct sources of inefficiency. First, there is involuntary unemployment since workers are paid in excess of their reservation wage. Second, firm scale is too large since the left hand side of (20) is necessarily positive. This second type of distortion does not appear to have received much attention in the literature. Its macroeconomic implications certainly warrant further investigation.

V. Conclusion

Efficiency wage theory has been developed to help explain involuntary unemployment. This theory can be seen as a step towards construction of a microfoundation for Keynesian macroeconomics. The implications of efficiency wage theory not only may justify the use of traditional macroeconomic policy instruments, but may also call for other types of policy intervention targeted more specifically at resolution of the relevant microeconomic problems. For example, wage subsidies financed by lump-sum taxes are an effective type of intervention in an economy where an efficiency wage equilibrium prevails, because such subsidies can raise the value of employment to the workers and, thus, reduce the need for unemployment as a worker discipline device.

Performance bonds are an alternate means of handling the worker moral hazard problem arising from imperfections in effort observability, which is the basis for efficiency wage theory. In actuality, employment contracts seem to be a hybrid of efficiency wage and implicit bonding which is observed in the form of upward sloping wage profiles steeper than worker productivity profiles (Medoff and Abraham, 1980; Lazear and Moore, 1984). The use of performance bonds, however, creates its own problem of employer moral hazard. Employer reputations can overcome this moral hazard problem when the market provides firms with sufficient rents from their reputations as honest employers. In this paper, we have shown that such rents can be reconciled with the free entry condition in competitive markets if firms pay entry fees and follow employment paths which signal their unobservable characteristics. However, the fact that there has to be rents to maintain the bonding equilibrium implies that product prices have to be higher than the marginal production costs. Though there is no involuntary unemployment in the

bonding equilibrium, this wedge is indicative of a product market distortion. Note that even when worker-posted bonds are circumscribed by unobservability of firm size, employer reputation is necessary as long as a part of workers' compensation takes the form of deferred payments, such as severance payments or pension plans. Thus, even in the absence of explicit bonding, reputation rents may exist in equilibrium, and consequently product market distortions will result.

Firm entry fees in bonding or deferred payment equilibria can be seen as a reputation-building investment to which reputation rents are attributed as normal profits. When the size of the firm is observable, a natural way of making such an investment is to pay signing bonuses to the workers employed in the period of entry. When the size of the firm is not observable, this kind of investment is not viable. Hence, entry fees will appear in many disparate forms, such as payments to third parties. The role of such expenditures is to convince workers about the firm's characteristics and incentives. This role is very similar to the role ascribed to advertising in signalling product quality in markets where product quality cannot be observed at the time of purchase. [Klein and Leffler, 1981; Milgrom and Roberts, 1986]. As such, it constitutes an alternative explanation of uninformative advertising itself.

Existence of public information regarding workers' employment history can give rise to worker reputations. This may increase the cost of shirking for workers and partially mitigate the worker moral hazard problem. However, workers must still be provided with incentives to prefer their reputations as conscientious workers to the short term benefits of shirking. As in the case of employer reputation, worker reputation cannot remove the unobservability distortion altogether.

The bonding equilibrium we have described is charaterized by underemployment, when there are some workers not under contract to any firm. That is, more demand could be satisfied at a lower price if it were not necessary that firms earn rents to sustain their reputations. In this case, just as in the case of efficiency wage equilibrium, policy intervention may be useful to attain Pareto improvements. In particular, demand subsidies can help raise production and employment to their optimal levels. However, there may be certain policies relevant to addressing the underemployment in bonding equilibrium which are irrelevant to addressing this problem in efficiency wage equilibrium, owing to the respective presence or absence of the employer moral hazard problem. One such policy is the imposition of legal penalties to discourage bankruptcy, thereby making it more costly for firms to expropriate performance bonds.

We conclude by calling attention to the observation that reputation may be but one mechanism to resolve the worker and firm moral hazard problems. We suspect that there are a multitude of formal and informal labor market institutions which address such problems. We also suspect that the concomitant distortions are manifested in a variety of ways. Comparing the relative efficiencies of such institutions in dealing with the moral hazard problems of the labor market is certainly quite a worthwhile line of research.

Notes

[1]The fact that actual labor contracts rarely call for the explicit posting of performance bonds would appear to supply the efficiency wage theory advocates with some rather potent ammunition.

[2]Studies of the implications of credit constraints can be found in Eaton and White (1982) and Akerlof and Katz (1986).

[3]It has been argued that, at least when capital markets are perfect, the employer moral hazard problem associated with bond expropriation can be overcome by using third party repositories for holding the performance bonds and by designating parties other than the firm as the recipients of forfeited performance bonds, when the firm fires workers for shirking. [Again, see Carmichael (1985).] In other words, the firm has no incentive to deliberately mislabel workers as shirkers, since it does not profit when these workers are fired. Our view is that such schemes may indeed raise the cost of bond expropriation to the firm, but do not rule out the incentives for bond expropriation entirely. For example, under Carmichael's scheme the firm could credibly threaten a worker with dismissal unless the firm receives some side payment from the worker. This extortion threat is analagous to the threat of bond expropriation. Alternatively, the firm could collude with the recipients of forfeited performance bonds and thereby find it profitable to deliberately mislabel workers as shirkers. A third possibility, when the firm is not the recipient of forfeited performance bonds, is that it is the workers who collude with these recipients, in order to defeat the incentives ascribed to performance bonds in the first place.

In this paper we ignore the possibility of contracts with third parties to rule out the added complication that the analysis of such contracts entails. We note that to the extent that bonding is implicit (see note 6), this restriction to bilateral contracts does not appear to be at odds with actual labor contracts.

[4]It is this worker inability to compute firm rents, because of inadequate information, which we feel to be the best explanation for the absence of performance bonds in actuality.

[5]The recent literature on implicit contracts with asymmetric information suggests strongly that workers may have inadequate information to compute firm rents even when firm employment is observable. See Hart (1983) for a survey of this literature. Indeed, this idea may provide an interesting way to connect the implicit contracts under asymmetric information and efficiency wage literatures.

[6]With efficiency wages, it is not the compensation today that deters workers from shirking today, but rather rents embedded in the future compensation scheme which acts as an incentive device to motivate workers to put forth effort today. These rents could equally well be embedded in pension, severance, or deferred wage payments. Moreover, when retirement is introduced into the model, some minimum level of deferred compensation is necessary to resolve the worker moral hazard just prior to the date of retirement.

[7]If the contract is not acceptable to a sufficient number of workers then it is null and void. In this case the firm must wait till the next period to offer a new contract but any worker who accepted the contract is free to seek employment elsewhere in the current period.

[8]In fact, we assume that the firm does not use the observation of $f[(1-k)L]$ in the detection of individual worker shirking. This assumption is reasonable for small values of k, but breaks down for k near 1. Obviously, when $k=1$ the firm can infer from observation of its output that essentially all of its workforce shirked. In the equilibrium described below $k=0$ and such a problem does not arise.

[9]Actually, we do not have well defined subgames here because the firm's information set about a worker that has not been detected shirking contains two nodes. That is, the firm can't distinguish between a worker who has put forth effort and a worker who shirked but was not detected. But since the firm is never able to make this distinction in future periods, it acts as if all such workers have a clean slate. Thus, what we really mean is that we wish to construct perfect equilibria of this game-form.

[10]For a more general analysis of subgame perfect equilibria in a similar contract game, see MacLeod and Malcomson (1987). Their model differs from ours in several respects. Among these are: (i) they assume detection occurs with certainty, (ii) they assume that production is additively separable in the labor input, and (iii) they ignore the possibility of turnover. The most important difference, however, is that they allow for the possibility of a perfectly enforceable severance tax, to deter workers from shirking and then being fired. In the presence of labor turnover for reasons other than shirking, it is hard to imagine that workers would agree beforehand to such a severance tax. It is also hard to believe that such taxes are enforceable ex post if these taxes exceed the amount that the worker has already paid into a severance fund.

[11]The reader will note that we are using the "Strong Law of Large Numbers" applied to the continuum when we conclude that θkL is the number of workers who are detected shirking. As has been pointed out by Feldman and Gilles (1985) among others, this is not valid, in general. They offer several suggestions to remedy this problem. For the purpose of our paper we adopt their second remedy, which is to weaken the independence condition. That is, the condition that the random variable denoting worker β being detected shirking is independent of the random variable denoting worker β' being detected shirking. Under the assumption that a worker can't observe the decision to shirk or not by other workers before he faces this decision himself, this remedy does not appear to create any difficulties for our paper.

[12]Note that we assume being detected shirking and being separated for reasons other than shirking are independent events. We also assume that when the intersection of these events occurs, the worker is treated as a shirker. Because this intersection has nonzero probability, our asset equations appear somewhat different than the analogous conditions in continuous time formulations, such as those given by Shapiro and Stiglitz (1984).

[13]When the firm has a precommitment capability, it may be reasonable for the punishment to be milder than having to resort to the no trade equilibrium. For example, if the firm can earn nonnegative profits by paying efficiency wages, then it could be that the punishment is merely for workers to not accept any contract with bonding but to allow the firm to operate using efficiency wages. The impact of such a milder punishment is that the right hand side of condition (7) becomes π/L^e, where π is the per period profit in the punishment equilibrium. This will have no impact on the argument that the equilibrium contract is a full bonding contract, but because the NEC is made tighter when the punishment is weaker, equilibrium firm scale may be reduced.

[14]The reason why the NEC is made weaker when $s > 0$ is as follows. Each worker hired by the firm must be paid his reservation wage plus an increment with expected discounted present value which, in conjunction with the performance bond, is sufficient to deter the worker from shirking. When the firm has labor turnover, the expected discounted number of workers hired per job is $(1+r)/(1+r-s) > 1$. Thus, the total discounted present value cost per job in excess of the worker's reservation wage is the product of the expected discounted present value of the wage increment and $(1+r)/(1+r-s)$. It is this second factor which explains why the discounted present value of the cost per job decreases by more than the increase in the value of the bonds, when the firm shifts its contract away from efficiency wages and towards performance bonds.

[15]The equilibrium value of L^e cannot lie on the rising portion of the average product curve. Hence, there is a unique PBEE in this case.

[16]There is another reason, apart from simplicity, for assuming that the unknown parameter is a multiplicative factor. As in Section II, we would like the market to be able to observe L and $f((1-k)L)$ in order to infer k and thereby to detect whether the firm has expropriated any performance level. Thus m is best interpreted as the firm's internal price of the product.

[17]The entry fee $C(m)$ may be paid to workers employed in the first period as a signing bonus. In this case the signing bonus per worker is $C(m)/L_1(m)$.

[18]It is assumed that all firms of the same type play the same strategy. In our model, this is not a restrictive assumption.

[19]The strategy space for each firm is the set of pairs of nonnegative entry fees and nonnegative sequences of employment.

[20]As suggested by Lazear (1981), deferred payments may be used to provide effort incentives for workers who approach the end of their job tenures. Lazear, however, models the firm moral hazard problem by treating the cost of expropriation on a per worker basis. These costs are exogenously specified and vary across firms. Lazear does not link these costs to the building of firm reputations.

[21]As in the case of performance bonds discussed in section II, the NEC is weakened by increasing the severance payment and lowering the constant wage along the NSC, holding employment fixed. Thus, there is no reason for firms to pay higher wages than w^e accompanied by a lower severance payment.

[22]In this case the entry fee cannot be paid to workers since employment is not observable.

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Appendix

Proof of Lemma 2: The constraint set defined by (12i) and (12ii) is non-empty since $\hat{\sigma}$ defined in Lemma 1 is a feasible strategy. Since $\pi(m,\sigma(m))$ is continuous in $\sigma(m)$ (we employ the sup norm topology on strategies) and the left-hand sides of (12i) and (12ii) are continuous in the relevant truncation of the strategy $\sigma(m)$, each constraint delimiting the constraint set is closed. Thus the constraint set itself is closed since it is the intersection of closed sets. Since the present discounted value of profits is bounded above by $(\frac{1+r}{r})(pf(L^*)-w_rL^*)$ we can restrict $C(m)$, the entry fee, to a bounded interval. Since the right-hand side of (12i) is bounded uniformly in $\tilde{m}$, we can restrict $L_1(m)$ to a bounded interval. Then, applying an inductive argument, we can restrict $L_t(m)$ to a bounded interval, since the right-hand side of (12ii) is uniformly bounded in $\tilde{m}$ and since by the induction hypothesis, $\sum_{h=1}^{t-1} \frac{p\tilde{m}f(L_h(m)) - w_rL_h(m)}{(1+r)^{h-1}} - C(m)$ is bounded below.

Moreover, we can take this lower bound to be uniform since

$$\sum_{h=1}^{t-1} \frac{p\tilde{m}f(L_h(m)) - w_rL_h(m)}{(1+r)^{h-1}} - C(m) < \frac{1}{(1+r)^{t-1}} \left[\frac{pf(L^*) - w_rL^*}{r}\right]$$ cannot be

optimal. Hence, we can take $L_t(m)$ to be bounded above uniformly in t and m. Thus there exists a bounded sequence of feasible strategies such that the associated sequence of profits converges to the supremum of profits over all feasible strategies. This bounded sequence of feasible strategies necessarily has a convergent subsequence whose limit is feasible and, by the continuity of the objective function in $\sigma(m)$, must in fact be optimal. ||

Proof of Lemma 3: The constraints given by (12i) and (12ii) are the same for each m. Take $m_1 < m_2$. Then, $\rho(m_1,z) = \pi(m_1,\sigma^*(m_1,z)) < \pi(m_2,\sigma^*(m_1,z)) \leq \pi(m_2,\sigma^*(m_2,z)) = \rho(m_2,z)$, where the first inequality holds because π is increasing in m and the second inequality holds because $\sigma^*(m_2,z)$ is optimal for type m_2 firms given z while $\sigma^*(m_1,z)$ is feasible. This shows $\rho(z)$ is increasing in m.

Let $m_\lambda = \lambda m_1 + (1-\lambda)m_2$ where $0 \leq \lambda \leq 1$. Then, $\rho(m_1,z) \geq \pi(m_1,\sigma^*(m_\lambda,z))$ and $\rho(m_2,z) \geq \pi(m_2,\sigma^*(m_\lambda,z))$. Hence $\lambda\rho(m_1,z) + (1-\lambda)\rho(m_2,z) \geq \lambda\pi(m_1,\sigma^*(m_\lambda,z)) + (1-\lambda)\pi(m_2,\sigma^*(m_\lambda,\dot{z})) = \pi(m_\lambda,\sigma^*(m_\lambda,z)) = \rho(m_\lambda,z)$. This shows $\rho(z)$ is convex in m.

Suppose $\rho(m_c,z) = \pi(m_c,\sigma^*(m_c,z)) > 0$. Then for $\tilde{m}$ near to but less than m_c, $\pi(\tilde{m},\sigma^*(m_c,z)) > 0$ as well. Thus by (9), $B_f L_t^*(m_c)$

$$\leq \sum_{h=t+1}^{\infty} \frac{p\tilde{m}f(L_h^*(m_c)) - w_r L_h^*(m_c)}{(1+r)^{h-t}}$$ for all t and $\tilde{m}$ near to but less than m_c.

Let $L^*(\tilde{m})$ solve $\max_{L\geq 0} p\tilde{m}f(L) - w_r L$. Since $\pi(\tilde{m},\sigma^*(m_c,z)) > 0$ it follows that $L^*(\tilde{m}) > \bar{L}$. Let $\bar{L}(m_c) = \sup_t L_t^*(m_c)$. Since $B_F L_t^*(m_c)$

$$\leq \sum_{h=t+1}^{\infty} \frac{p\tilde{m}f(L_h^*(m_c)) - w_r L_h^*(m_c)}{(1+r)^{h-t}} \leq \frac{p\tilde{m}f(L^*(\tilde{m})) - w_r L^*(\tilde{m})}{r} < B_F L^*(\tilde{m}),$$ it follows

that $\bar{L}(m_c) \leq L^*(\tilde{m})$ for $\tilde{m}$ near to but less than m_c. Since $p\tilde{m}f(L_h^*(m_c)) - w_r L_h^*(m_c) \leq p\tilde{m}f(\bar{L}(m_c)) - w_r \bar{L}(m_c)$ for all h,

$$B_F L_t^*(m_c) \leq \frac{p\tilde{m}f(\bar{L}(m_c)) - w_r \bar{L}(m_c)}{r} \leq B_F \bar{L}(m_c) - \frac{p(m_c-\tilde{m})f(\bar{L}(m_c))}{r}.$$ For t such

that $L_t^*(m_c)$ is sufficiently close to $\bar{L}(m_c)$ this is not possible. Hence $\rho(m_c,z) = 0$. ||

Proof of Theorem 1: Let $L^{\infty}[m_c,1]$ be the set of all almost everywhere bounded, measurable functions on $[m_c,1]$ endowed with the sup norm, i.e., for $f \in L^{\infty}[m_c,1]$, $\|f\|_{\infty} = \inf\{N: \mu\{m: f(m) > N\} = 0\}$ where μ is Lebesque measure. $L^{\infty}[m_c,1]$ is a Banach Space. Let $Z[m_c,1] = \{z \in L^{\infty}[m_c,1]$: z is nondecreasing, convex, $z(m_c) = 0$, and $z(1) \leq (\frac{1+r}{r})[pf(L^*)-w_r L^*]\}$. $Z[m_c,1]$ is obviously a convex set. We will also show that it is closed and sequentially compact in the sup norm topology. Since each $z \in Z[m_c,1]$ can be trivially extended to a function, z^e, defined on $[\underline{m},1]$ by requiring $z^e(m) = 0$ for $m < m_c$ and $z^e(m) = z(m)$ for $m \geq m_c$, the function ρ composed with this extension function induces a map from $Z[m_c,1]$ into itself: $z \rightarrow \rho(z^e)$. This map is continuous and hence by the Bohnenblust and Karlin fixed point theorem (see Friedman (1977), p. 162) this map has a fixed point.

We first show $Z[m_c,1]$ is closed. Let (z_q) be a convergent sequence in $Z[m_c,1]$ and let z be the pointwise limit of this sequence. Take $m_c \leq m_1 < m_2 \leq 1$. Then since for each q, $z_q(m_1) \leq z_q(m_2)$, we have $z(m_1) = \lim_{q\to\infty} z_q(m_1) \leq \lim_{q\to\infty} z_q(m_2) = z(m_2)$. Hence z is nondecreasing. Now take $m_\lambda = \lambda m_1 + (1-\lambda)m_2$ with $0 \leq \lambda \leq 1$. Again we have for each q $\lambda z_q(m_1) + (1-\lambda)z_q(m_2) \geq z_q(m_\lambda)$. Hence $\lambda z(m_1) + (1-\lambda)z(m_2) = \lambda \lim_{q\to\infty} z_q(m_1) + (1-\lambda) \lim_{q\to\infty} z_q(m_2) = \lim_{q\to\infty}[\lambda z_q(m_1)+(1-\lambda)z_q(m_2)] \geq \lim_{q\to\infty} z_q(m_\lambda) = z(m_\lambda)$. Hence z is convex. Since $z_q(m_c) = 0$ for all q, $\lim_{q\to\infty} z_q(m_c) = z(m_c) = 0$. Finally, since $z_q(1) \leq (\frac{1+r}{r})[pf(L^*)-w_r L^*]$ for all q, $\lim_{q\to\infty} z_q(1) = z(1) \leq (\frac{1+r}{r})[pf(L^*)-w_r L^*]$. Thus $Z[m_c,1]$ is complete. Since $Z[m_c,1]$ is a complete subset of a metric space, it is closed (see Royden, 1968, p. 138).

Now we show $Z[m_c,1]$ is sequentially compact. Let M be a countable dense subset of $[m_c,1]$ and order elements in M so that m_i denotes the ith element in M. Set $m_1 = 1$. Consider an arbitrary sequence $(z_q) \subset Z[m_c,1]$. We are to show that (z_q) necessarily has a convergent subsequence. Since $(z_q(m_1)) = (z_q(1))$ is a bounded sequence in R^1, it necessarily has a convergent subsequence by the Bolzano-Weierstrass Theorem. Then since every subsequence of a convergent sequence converges, it follows by induction that there exists a subsequence of (z_q), denote this subsequence by (z_q^k), such that $(z_q^k(m_i))$ converges for $i = 1,\ldots,k$, where we note that $(z_q(m_k))$ and hence any subsequence of $(z_q(m_k))$ is bounded. Since this is true for all k, it follows that there is a subsequence of (z_q), denote this subsequence by (z_q^∞), such that $(z_q^\infty(m_i))$ converges for all $m_i \in M$. Now take an arbitrary $m \in [m_c,1]$. Then $(z_q^\infty(m))$ converges. To see this note that it is necessarily true for $m = 1$ since $m_1 = 1$. If $m = m_c$, then $z_q^\infty(m_c) = 0$ for all q and consequently $(z_q^\infty(m_c))$ converges to 0. Finally, take $m_c < m < 1$ and let z^* be an accumulation point of $(z_q^\infty(m))$. Let $(m_{i_h}) \subset M$ such that $m_{i_h} \uparrow m$. Then since $z_q^\infty(m_{i_h}) \leq z_q^\infty(m)$ fo each q and h, $\lim_{h\to\infty} \lim_{q\to\infty} z_q^\infty(m_{i_h}) \leq z^*$. Since z_q^∞ is convex, $z_q^\infty(m) \leq [\frac{1-m}{1-m_{i_h}}]z_q^\infty(m_{i_h}) + [\frac{m-m_{i_h}}{1-m_{i_h}}]z_q(1)$. Hence

$z^* \leq \lim_{h\to\infty} \lim_{q\to\infty} \{[\frac{1-m}{1-m_{i_h}}]z_q^\infty(m_{i_h}) + [\frac{m-m_{i_h}}{1-m_{i_h}}]z_q^\infty(1)\}$. Thus z^* is unique and $(z_q^\infty(m))$ converges.

To show that the composition of ρ with the extension function is continuous, take $z_1,z_2 \in Z[m_c,1]$ such that $\|z_1-z_2\|_\infty < \varepsilon$. Then if $\sigma^*(m,z_1^e)$ is altered by increasing the entry fee by ε without changing the employment path, the resulting altered strategy satisfies all the constraints in (12) for the profit assignment function z_2^e. Thus $\rho(m,z_1^e) - \varepsilon \leq \rho(m,z_2^e)$.

Similarly, $\rho(m,z_2^e) - \varepsilon \leq \rho(m,z_1^e)$. This is true uniformly in m. Thus $\|\rho(z_1^e) - \rho(z_2^e)\| < \varepsilon$. ‖

<u>Proof of Lemma 4</u>: Since σ^* is a Nash equilibrium, $\sigma^*(\tilde{m}) = \pi(\tilde{m},\sigma^*(\tilde{m})) \geq \pi(\tilde{m},\sigma^*(m))$ for all $m,\tilde{m} \in [m_c,1]$. It follows that given σ^*, $\sigma^T(\sigma^*(m))$ satisfies (12i) and satisfies (12ii) for $t = 2,\ldots,T$, since $\sigma(m)$ satisfies these constraints and $\sigma^T(\sigma^*(m))$ coincides with $\sigma^*(m)$ over these periods. If $\tilde{p}mf(L_T^*(m)) - w_r L_T^*(m) \geq rB_f L_T^*(m)$, then for $t > T$

$$\sum_{h=1}^{t} \frac{\tilde{p}mf(L_h^T(\sigma^*(m))) - w_r L_h^T(\sigma^*(m))}{(1+r)^{h-1}} + \frac{B_f L_T^*(m)}{(1+r)^{t-1}} \leq \pi(\tilde{m},\sigma^T(\sigma^*(m))) \text{ (see}$$

equation (9) and the prior discussion). While if

$\tilde{p}mf(L_T^*(m)) - w_r L_T^*(m) < rB_F L_T^*(m)$, then for $t > T$

$$\sum_{h=1}^{t} \frac{\tilde{p}mf(L_h^T(\sigma^*(m))) - w_r L_h^T(\sigma^*(m))}{(1+r)^{h-1}} + \frac{B_F L_T^*(m)}{(1+r)^{t-1}} < \sum_{h=1}^{T} \frac{\tilde{p}mf(L_h^*(m)) - w_r L_h^*(m))}{(1+r)^{h-1}}$$

$$+ \frac{B_F L_T^*(m)}{(1+r)^{T-1}} \leq \sigma^*(\tilde{m}).$$ Hence, $\sigma^T(\sigma^*(m))$ satisfies (12i) and (12ii). ‖

<u>Proof of Lemma 5</u>: Since $\pi(m,\sigma^*(m)) = \sigma^*(m)$, it follows that

$$B_F L_t^*(m) \leq \sum_{h=t+1}^{\infty} \frac{pmf(L_h^*(m)) - w_r L_h^*(m)}{(1+r)^{h-t}}$$ for each t (again see equation (9)

and the prior discussion). There are two cases to consider. First, $L^*(m) < \operatorname{argmax}_L pmf(L) - w_r L$. Then $L^*(m)$ satisfies $pmf(L^*(m)) - w_r L^*(m) = rB_F L^*(m)$. It is straightforward to show that in this case $pmf(L_t^*(m)) - w_r L_t^*(m)) \geq rB_F L_t^*(m)$ when $L_t^*(m) \geq \bar{L}$. The argument is the same as the one given in the proof of Lemma 3 to show $\rho(m_c,z) = 0$. Hence $L_t^*(m) \leq L^*(m)$ is immediate in this case. Second, $L^*(m) = \operatorname{argmax}_L pmf(L) - w_r L$. In this case, if $L_T^*(m) > L^*(m)$ for any T, then an alternate feasible strategy can be constructed with the same entry fee and same employment

levels for the first T - 1 periods and with stationary employment equal to $L^*(m)$ in all subsequent periods. The feasibility of this alternate strategy follows from the same argument as given in the proof of Lemma 4, by noting that in lowering employment in period T from $\ell_T^*(m)$ to $L^*(m)$ the period T constraints in (12ii) are all weakened. Since this alternate strategy yields greater profits than $\sigma^*(m)$, $\sigma^*(m)$ could not be a best response to σ^*. This is a contradiction. ||

Proof of Theorem 2: Without loss of generality take $m_c \leq m < \tilde{m} \leq 1$. Given $\sigma^*(m)$, we will construct an alternate feasible strategy which provides a type m firm with less profit than $\sigma^*(m)$ and provides a type $\tilde{m}$ firm with greater profit than $\sigma^*(m)$.

When $m = m_c$ if $c^*(m_c) = L_t^*(m_c) = 0$ for all t, the result is trivial since the strategy $\hat{\sigma}$, defined in Lemma 1, is feasible and gives the type $\tilde{m}$ firm greater profits than it would earn by staying out.

Having dispensed with this case we assume without loss of generality that $pmf(L_t^*(m)) - w_r L_t^*(m) > 0$ infinitely often, by invoking Lemma 4 if necessary. In each period where profits are positive, the shape of the production function and Lemma 5 imply that $pmf'(L_t^*(m)) - w_r \geq 0$. Denote the first such period at t_1, the second such period as t_2, etc. We construct the alternate strategy as follows. Denote the alternate strategy by $\sigma^a = (C^a, (L_t^a))$. Then $C^a = C^*(m) + dC + \varepsilon$, where dC and ε are both positive but small. ε is further specified below. If $pmf(L_t^*(m)) - w_r L_t^*(m) \leq 0$, then $L_t^a = L_t^*(m)$. If not, then $L_{t_q}^a = L_{t_q}^*(m) + dL_{t_q}$ where dL_{t_q} is defined inductively by

$$dL_{t_1} = \frac{dC(1+r)^{t_1-1}}{[pmf'(L^*_{t_1}(m)) + B_F - w_r]} \qquad \text{and}$$

$$dL_{t_q} = \frac{dC - \sum_{h=1}^{q-1} [pmf'(L_{t_h}(m)) - w_r]/dL_{t_h}(1+r)^{t_h-1}}{[pmf'(L^*_{t_q}(m)) + B_F - w_r](1+r)^{1-t_q}} .$$

Observe that by construction

$$\sum_{h=1}^{q} \frac{[pmf'(L^*_{t_h}(m)) - w_r]dL_{t_h}}{(1+r)^{t_h-1}} + \frac{B_F dL_{t_q}}{(1+r)^{t_q-1}} = dC.$$

By a straightforward induction argument, $0 < dL_{t_q} < \frac{dC}{B_F}$. Hence,

$$\sum_{h=1}^{\infty} \frac{[pmf'(L^*_{t_h}(m)) - w_r]L_{t_h}}{(1+r)^{t_h-1}} = dC.$$

Let dm be defined by

$$dm = \frac{dC}{\inf_t [\sum_{h=t+1}^{\infty} pmf(L^*_h(m))/(1+r)^{h-t}]} .$$

For $\hat{m} \geq m + dm$ we have

$$\sum_{h=t+1}^{\infty} \frac{p\hat{m}f(L^a_h) - w_r L^a_h}{(1+r)^{h-t}} > \sum_{h=t+1}^{\infty} \frac{p\hat{m}f(L^*_h(m)) - w_r L^*_h(m)}{(1+r)^{h-t}}$$

$$\geq \sum_{h=t+1}^{\infty} \frac{pmf(L^*_h(m)) - w_r L^*_h(m)}{(1+r)^{h-t}} + dC \geq B_F[L^*_t(m) + \frac{dC}{B_F}] \geq B_F L^a_t.$$

Thus, no such firm will expropriate bonds under this alternate policy. If ε is set so that $\sum_{h=1}^{\infty} pdmf'(L_{t_h}(m))dL_{t_h}/(1+r)^{t_h-1} = \varepsilon$, then it follows that for all $\hat{m} \leq m + dm$ and for all q,

$$\sum_{h=1}^{q} \frac{[\hat{p}mf'(L^*_{t_h}(m)) - w_r]dL_{t_h}}{(1+r)^{t_h-1}} + \frac{B_F dL_{t_q}}{(1+r)^{t_q-1}} <$$

$$\sum_{h=1}^{q} \frac{pdmf'(L^*_{t_h}(m))dL_{t_h}}{(1+r)^{t_h-1}} + dC = \varepsilon + dC.$$

Hence, the alternate policy is feasible.

Finally, note that the change in profits for a type m firm in adopting the alternate policy is

$$\sum_{h=0}^{\infty} \frac{[pmf'(L^*_{t_h}(m)) - w_r]dL_{t_h}}{(1+r)^{t_h-1}} - [dC+\varepsilon] = -\varepsilon < 0$$

while the change in profits for a type $\tilde{m}$ in adopting the alternate policy is

$$\sum_{h=0}^{\infty} \frac{[p\tilde{m}f'(L^*_{t_h}(m)) - w_r]dL_{t_h}}{(1+r)^{t_h-1}} - [dC+\varepsilon] =$$

$$\sum_{h=0}^{\infty} \frac{p(\tilde{m}-m)f'(L^*_{t_h}(m))dL_{t_h}}{(1+r)^{t_h-1}} - \varepsilon > 0,$$

as long as $\tilde{m} - m > dm$. ||

<u>Proof of Theorem 3</u>: $\pi(m,\sigma^*(m)) \geq \pi(m,\hat{\sigma})$ and $\pi(m_c,\hat{\sigma}) \geq \pi(m_c,\sigma^*(m))$ since σ^* is a Nash equilibrium. These inequalities imply that

$$\sum_{t=1}^{\infty} p(m-m_c) \frac{f(L^*(m))}{(1+r)^{t-1}} \geq \sum_{t=1}^{\infty} p(m-m_c) \frac{f(\overline{L})}{(1+r)^{t-1}} .$$

Hence $\overline{L}^*(m) \equiv \sup(L_t^*(m)) \geq \overline{L}$. From Lemma 4, it follows that

$$\sum_{h=t+1}^{\infty} \frac{pmf(L_h^*(m)) - w_r L_h^*(m)}{(1+r)^{h-t}} \geq \frac{pmf(L_t^*(m)) - w_r L_t^*(m)}{r}$$

for all t. In fact,

$$\sum_{h=t+1}^{\infty} \frac{pmf(L_h^*(m)) - w_r L_h^*(m)}{(1+r)^{h-t}} \geq \max_{\tau \leq t} \left[\frac{pmf(L_\tau^*(m)) - w_r L_\tau^*(m)}{r}\right]$$

since

$$\sum_{h=\tau}^{\infty} \frac{pmf(L_h^*(m)) - w_r L_h^*(m)}{(1+r)^{h-\tau}} = \sum_{h=\tau}^{t} \frac{pmf(L_h^*(m)) - w_r L_h^*(m)}{(1+r)^{h-\tau}}$$

$$+ \frac{1}{(1+r)^{t-\tau}} \sum_{h=t+1}^{\infty} \frac{pmf(L_h^*(m)) - w_r L_h^*(m)}{(1+r)^{h-t}} .$$

Let t_1 be the smallest t such that $L_t^*(m) \geq \overline{L}$. If $\overline{L}^*(m) < L^*(m)$ then it follows that there exists an $\hat{m}$, $m_c < \hat{m} < m$, such that

$$B_F L_t^*(m) \leq \sum_{h=t+1}^{\infty} \frac{\hat{p}mf(L_h^*(m)) - w_r L_h^*(m)}{(1+r)^{h-t}}$$

for all $t \geq t_1$. Hence, there exists an alternate feasible strategy which yields greater profits for a type m firm than $\sigma^*(m)$. The argument is the same as the one given in the proof of Theorem 2. Thus $\overline{L}^*(m) = L^*(m)$.

We must show that $\overline{L}^*(m)$ is the only accumulation point of $(L_t^*(m))$. Since

$$\sum_{h=t+1}^{\infty} \frac{pmf(L_h^*(m)) - w_r L_h^*(m)}{(1+r)^{h-t}}$$

$$\leq \frac{1}{r} \{ (\frac{r}{1+r}) [pmf(L_{t+1}^*(m)) - w_r L_{t+1}^*(m)] + (\frac{1}{1+r}) [pmf(\overline{L}^*(m)) - w_r \overline{L}^*(m)]\}$$

it follows from the results of the previous paragraph that

$$\max_{\tau \leq t} [pmf(L_\tau^*(m)) - w_r L_\tau^*(m)] - [pmf(L_{t+1}^*) - w_r L_{t+1}^*(m)] \leq$$

$$\frac{1}{r} \{pmf(\overline{L}^*(m)) - w_r \overline{L}^*(m)] - \max_{\tau \leq t} [pmf(L_\tau^*(m)) - w_r L_\tau^*(m)]\}.$$

Since the right-hand side of this inequality converges to 0 as t goes to infinity, it follows that there is a unique accumulation point of $(L_t^*(m))$. ||

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